Efficient Robust Control of Mixed Platoon for Improving Fuel Economy and Ride Comfort

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***Abstract*—The emergence of connected and automated vehicle technology has improved the operational efficiency of mixed traffic systems. This paper studies a two-tier trajectory optimization problem for mixed platooning to improve fuel efficiency, ride comfort, and operational safety during vehicle operations. The proposed model follows a two-tier control logic to plan the trajectory of platooning vehicles with three objectives, including minimizing fuel consumption, maximizing ride comfort, and enhancing the anti-disturbance performance of the platoon. The first is the planning tier, which aims to design the optimal trajectory for Connected and Automated Vehicles (CAVs) based on the optimal fuel consumption and comfort and obtain the expected acceleration curve of CAV. The second is the control tier, which aims to ensure the safe operation of platooning vehicles in the presence of uncertain disturbances in real time. Specifically, we propose a robust tube MPC control method, which dynamically adjusts the CAV acceleration according to the reference trajectory obtained by the planning tier to resist the effects of uncertain disturbances, and the tracking behaviour of Human-Driven Vehicles (HDVs) is offline solved by the robust optimal velocity model. Finally, we design simulation experiments to verify the effectiveness of the proposed two-tier optimization framework. The experimental results show the effectiveness and advantages of the two-tier framework in terms of fuel economy, ride comfort, and robustness against different noise disturbances.**

***Index Terms*—Vehicle Platoon, Connected and Automated Vehicles, Trajectory Optimisation, Fuel Economy, Ride Comfort, Tube Model Predictive Control**

I. INTRODUCTION

TRANSPORTATION is one of the significant sources of energy consumption and greenhouse gas emissions. According to statistics [1], in 2020, transportation accounted for 32% of the total energy consumption and 27% of the total emissions in the European Union, with a significant portion attributed to road transportation. Despite significant efforts to alleviate the burden on fuel economy, such as optimizing vehicle engines and improving road conditions, the total energy consumption of motor vehicles has continued to rise in recent years due to the continuously increasing number of motor vehicles. Moreover, vehicle ride comfort exhibits typical traffic flow characteristics, which affect the vehicle's lateral acceleration and increase consumer experience value [2], [3]. Therefore, vehicles' energy consumption and ride comfort should be carefully considered to improve the current traffic conditions.

The motion of vehicles plays a vital role in improving the transportation system, and it has greatly benefited from emerging Intelligent Connected Vehicle (ICV) technologies such as wireless communication [4], adaptive cruise control [5], and speed harmonization [6]. CAVs have demonstrated significant advantages in traffic safety, efficiency, and energy consumption, such as the cooperative platooning of multiple CAVs that can improve road capacity [7]–[9]. However, a transitional phase of mixed traffic systems must exist in real-life scenarios comprising CAVs and HDVs. By leveraging the physical information and interaction between neighbouring vehicles, CAVs can be considered mobile actuators to induce the directed motion of surrounding vehicles and thus control the driving behaviour of mixed traffic platoons. This approach can effectively enable the safe and efficient operation of mixed traffic systems. This paper first proposes a bi-objective trajectory planning model to improve the fuel economy and ride comfort of the mixed vehicle platoon system.

In a mixed-vehicle platoon system, CAVs typically serve as mobile leaders, influencing the motion of other vehicles. Vehicle-to-vehicle (V2V) communication is employed to transmit vehicle information. However, when CAVs track the trajectory of the preceding mobile leader, they often encounter random interference caused by carrier-to-interference, resulting in the control system's inability to meet physical requirements and leading to unstable platoon formation or even chain collisions [10]. This interference arises from the high mobility of vehicles, resulting in high Doppler frequency and inter-carrier interference [11]. Additionally, communication bandwidth constraints lead to delays in V2V transmission, contributing to communication delays and ultimately causing sensor measurement errors [12]. Therefore, CAVs in the platoon system need to develop distributed robust controllers resistant to interference to ensure adaptive control of platoon vehicles under noise interference. Accordingly, we propose a robust tube control method to ensure the real-time anti-

This research was supported in part by the National Key Research and Development Program of China under Grant No. 2022YFC3803700, in part by the National Natural Science Foundation of China under Grant No. U20A20155, Grant No. 52202391, Grant No. 62061130221, Grant No. 62173012. *(Corresponding author: Daxin Tian.)*

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interference property of vehicle tracking.

In summary, this study aims to establish a two-tier planning framework to optimize the motion trajectory of mixed vehicle platoons. The proposed objectives include minimizing fuel consumption, maximizing ride comfort, and improving the platoon's disturbance rejection capability. The first tier is CAV's optimal trajectory design, which proposes a bi-objective model to improve platoon fuel economy and ride comfort and obtain vehicle acceleration curves. The second tier is for real-time control purposes to ensure that the vehicle is anti-interference in trajectory tracking. Precisely, a robust tube controller will be placed on the CAV to dynamically adjust the vehicle's acceleration while the other HDVs perform trajectory tracking, the tracking behaviour of HDVs is obtained offline through the optimal speed model. Our contributions are summarized as follows.

- A novel two-tier optimization framework for mixed vehicle platooning is proposed to improve the platoon's fuel economy, ride comfort, and disturbance rejection. At the planning tier, we design a bi-objective model for CAVs to enhance the platoon's fuel economy and ride comfort and obtain the reference trajectory of the vehicles.
- To ensure the safety of the platoon vehicles in real-time control, we design a robust Tube MPC controller to adjust vehicle acceleration to resist uncertain disturbances dynamically. The robustness can achieve stable tracking of platoon vehicles and ensure the safety of vehicles.
- We conduct simulations to verify the proposed two-tier programming model. In addition, we also illustrate that the proposed method is superior to the other strategies in terms of anti-interference, fuel efficiency and ride comfort.

The remaining parts of this paper are organized as follows: Section II analyzes the related work. Section III provides the system framework of the two-tier trajectory planning problem. Section IV presents the CAV dynamics, HDV car-following model, and offline bi-objective trajectory planning model. The tube-based robust MPC controller is designed in Section V. Then, we conduct a series of simulation experiments in Section VI. Finally, conclusions are drawn, and future research directions are discussed in Section VII.

II. Literature Review

In recent decades, improving the fuel economy of vehicle platooning systems has become a significant research focus. Studies, such as that by reference [13], have found that formation driving can increase fuel consumption by approximately 5% compared to single-vehicle driving. Therefore, optimizing the trajectory planning of platooned vehicles is essential to improve fuel efficiency further. [9] has proposed a cooperative eco-driving model for mixed vehicle platoon to improve fuel consumption and reduce idle time. In [14], a speed planning algorithm is proposed for a platoon of trucks on a highway. This algorithm uses an integrated fuel time cost and dynamic programming strategy. The experimental results demonstrate that the vehicle platoon following this speed curve achieves better fuel efficiency. [15] investigated the gasoline consumption and gas emissions of CAVs in mixed traffic flows when passing through intersections and proposed a joint optimization framework of traffic signals and vehicle trajectories to reduce the environmental impact of these factors. [16] proposed an eco-driving strategy for mixed platoons of CAVs and HDVs, consisting of offline planning and online tracking using model predictive control. The strategy optimizes total fuel consumption by determining each vehicle's energy-efficient speed and gearshift references.

In addition to fuel economy, passenger comfort is an essential aspect of vehicle platooning systems, as it directly affects passenger satisfaction and safety [17]–[20]. [2] proposed an MPC-Based cruise adaptive control method for autonomous vehicles to enhance driving safety and comfort. The approach planned predicted vehicle control commands for the host vehicle within a prediction horizon and then computed a constrained optimization model in the finite time domain. [21] proposed a bi-objective optimization model to reduce fuel consumption and improve driver comfort in the mixed vehicle platoon. A fuzzy logic-based integrated sliding mode controller designed by [22] and applied to vehicle suspension. This controller can improve ride comfort by reducing vehicle vibrations. To enhance fuel efficiency and ride comfort, a bi-objective optimization strategy is suggested for the planning stage to achieve the optimal driving path of the vehicle offline.

Although tracking the optimal offline trajectory obtained by the planning layer can improve fuel efficiency in platoon vehicles, unexpected disturbances can significantly impact performance. Real-time trajectory optimization methods that minimize fuel consumption have been proposed to enhance robustness, considering uncertainties from other traffic participants [23], [24]. However, existing predictive models that provide information about other transport partners are limited to the short term, and the real-time trajectory optimization method may not be optimal as a result [25], [26]. Therefore, a real-time control method is developed for CAV to resist uncertain disturbances during trajectory tracking. [27] and [28] illustrated a two-tier/two-stage optimization framework to bridge the gap between offline planning and real-time control layers—the offline planning tier real-time traffic situation by controlling the CAV to follow a reference trajectory. [29] proposed a two-level optimal control algorithm that reduces the fuel consumption of a platoon composed entirely of CAVs. However, a transition phase must be established for a mixed traffic system containing CAVs and HDVs [30].

At the control tier, various control methods have been proposed to deal with the impact of uncertain disturbances on platooning, such as H_∞ control [31], Model Predictive Control (MPC) [32], Kalman filter [33] and sliding mode control [34]. In addition, distributed model predictive control has been widely applied in engineering practice. When the platoon system was subject to persistent disturbances, conventional nominal MPC could not solve the physical constraints under disturbance, leading to uncontrollable platoon systems. Linear quadratic control is a special case of MPC which can be effective in regulating linear systems with known dynamics and noise characteristics, they may struggle to handle nonlinearities and uncertain disturbances, limiting their applicability in complex real-world systems [35]. In the context of dealing with uncertain disturbances in platooning, the Kalman filter, while effective for state estimation, may face limitations in adequately addressing physical constraints under persistent disturbances [36]. Inspired by the tube-based model predictive control method proposed by Mayne and Langson, the control problem of a mixed vehicle platooning subject to environmental disturbances and modelling errors was investigated in [37]. They proposed a control method using the multiple integral observer technique and tube-based model predictive control and optimizing the control strategy gains using the particle swarm optimization algorithm. In [38], a tube-based discrete controller was designed, where the vehicle's control signal was determined jointly by feedforward and feedback control. [39] highlighted the utilization of the Tube Model Predictive Control (TMPC) approach in the TUM Autonomous Motorsport team's software stack. By incorporating a simplified friction-limited point mass model and a constraint-tightening strategy, the proposed Tube MPC method outperforms traditional tracking controllers. However, there still needs to be a heterogeneous platoon control method that can simultaneously address vehicle economy, comfort, and disturbance resistance within a two-tier optimization framework.

III. Trajectory Optimization of Mixed Platoon

A. System Architecture

In this study, we address the trajectory optimization problem for a mixed-vehicle platoon containing CAVs and HDVs. As shown in Figure 1, we model this integrated optimization problem into a two-tier model. First, at the planning tier, the desired trajectory for the mixed platoon is optimized by minimizing fuel economy and maximizing ride comfort. Secondly, the control tier is developed to robust tube MPC strategy to resist random perturbations and to provide real-time control. More specifically, utilizing the acceleration profile obtained from the planning stage, a robust adaptive controller will be installed on the CAVs to regulate the control signal and ensure safe operations adaptively.

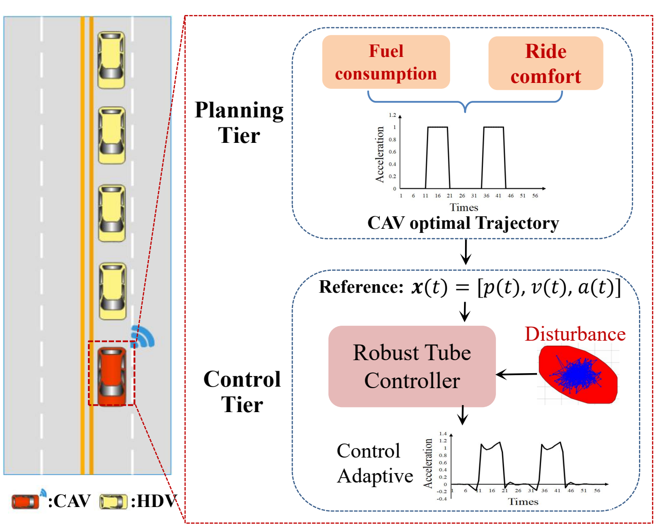


Fig. 1. The system scenario of the considered platoon.

As shown in Figure 1, we propose a fuel economy and ride comfort-oriented robust controller for the mixed vehicle platooning system, aiming to improve the fuel efficiency, comfort, and robustness of the platooning vehicles in uncertain environments. In the proposed two-tier optimization framework, the dynamics of the CAV are designed by the proposed robust model predictive control method, and the motion of the HDV is described by the optimal velocity model. In this paper, we only consider the longitudinal behaviour of the vehicle. Below, we first introduce the trajectory planning model of mixed vehicle formation.

TABLE I
Basic notations in trajectory planning

Symbol	Description
t	Time instant
t^0	Initial time
t^f	Terminal time
$\mathcal{I} = 1, \ldots, I$	The set of CAVs
$\mathcal{J} = 1, \ldots, J$	The set of HDVs
$p_i(t), v_i(t), a_i(t)$	Position, velocity and acceleration of CAV $i, i \in \mathcal{I}$
$p_j(t), v_j(t), a_j(t)$	Position, velocity and acceleration of HDV $j, j \in \mathcal{J}$
$u_i(t)$	The control variables of a CAV $i, i \in \mathcal{I}$

For HDVs, the car-following model OVM can drive the vehicle to reach the desired optimal speed. The acceleration profile $a_j(t)$ can be formulated in the following form.

$$a_j(t) = \eta \left[V_{opt}\left(\Delta p_j(t)\right) - v_j(t)\right], j \in \mathcal{J}. \tag{1}$$

where η is a sensitivity parameter, p_{j-1} is the position of vehicle $j-1$ and p_j is the position of vehicle j. $\Delta p_j(t) = p_{j-1}(t) - p_j(t)$ is the distance between vehicle j and $j-1$ and the speed function $V_{opt}\left(\Delta p_j(t)\right)$ is related to the distance $\Delta p_j(t)$. in this paper, we adopt the following speed function:

$$V_{opt}\left(\Delta p_j(t)\right) = \kappa_1 + \kappa_2 \tanh[C_1(\Delta p_j(t) - L) - C_2], \tag{2}$$

where κ_1, κ_2, C_1 and C_2 are the basic parameters. According to [40], the parameters of OVM are set in this paper: $\eta = 0.85\,\mathrm{s}^{-1}, \kappa_1 = 6.75\,\mathrm{m/s}, \kappa_2 = 7.91\,\mathrm{m/s}, C_1 = 0.13\,\mathrm{m}^{-1}, C_2 = 1.57$ and $L = 4.5\,\mathrm{m}$.

B. Trajectory Planning Model

In the mixed vehicle platoon, information such as position is shared between the CAVs via V2V. The state equation of CAV i is defined to the following form

$$\dot{p}_i = v_i(t), \quad \dot{v}_i = a_i(t), \quad \dot{a}_i = u_i(t), \quad i \in \mathcal{I}, \tag{3}$$

where $u_i(t)$ is the control variable, which is the derivative of the acceleration a_i. In order to obtain the optimal trajectory of all vehicles in the platoon, we define the state variable $\mathbf{X}(t)$ and the control variable $\mathbf{U}(t)$ of this platoon as

$$\begin{aligned} &\mathbf{X}(t) = [p_i(t), v_i(t), a_i(t), \cdots, p_I(t), v_I(t), a_I(t)], \\ &\mathbf{U}(t) = [u_i(t), \cdots, u_I(t)], i \in \mathcal{I}. \end{aligned} \tag{4}$$

Based on equation (4), the system dynamic function is

$$\dot{\mathbf{X}}(t) = [v_i(t), a_i(t), u_i(t), \cdots, v_I(t), a_I(t), u_I(t)], \tag{5}$$

where the acceleration $a_j(t)$ of HDVs are calculated by (1).

1) Objective Function: At the planning tier, we will optimize the two objectives of the vehicle platoon, the first is to minimize the total fuel consumption of the vehicle, and the second is to maximize the ride comfort. The first objective function is designed as

$$H_1(\mathbf{U}(t)) = \int_{t^0}^{t^f} \left[\sum_{i\in\mathcal{I}} F_i(t)\right] dt. \tag{6}$$

According to the fuel consumption model proposed by [41], the instantaneous acceleration and speed can estimate the fuel consumption rate of all vehicles, its expression is

$$F = \begin{cases} \alpha + \beta_1 P_T + \beta_2 m a^2 v, & \text{if } P_T \geq 0, \\ \alpha, & \text{if } P_T < 0, \end{cases} \tag{7}$$

among that α, β_1 and β_2 are constants, and $P_T(t)$ is the engine power of the vehicle driveline which is calculated by

$$P_T = \max\left\{0, \tau_1 v + \tau_2 v^2 + \tau_3 v^3 + mav\right\}. \tag{8}$$

where τ_1, τ_2 and τ_3 are constants, m is the weight of the vehicle. All parameters in $H_1(\mathbf{U}(t))$ are taken from [41] and $\alpha = 0.666\,\mathrm{mL/s}$, $\beta_1 = 0.072\,\mathrm{mL/kJ}$, $\beta_2 = 0.0344\,\mathrm{mL}/\left(\mathrm{kJ}\cdot\mathrm{m/s^2}\right)$, $\tau_1 = 0.269\,\mathrm{kN}$, $\tau_2 = 0.0171\,\mathrm{kN/(m/s)}$, $\tau_3 = 0.000672\,\mathrm{kN/(m/s)^2}$, $m = 1680\,\mathrm{kg}$.

To improve the longitudinal ride comfort of the mixed platoon, we limit frequent changes in engine speed by reducing changes in acceleration (i.e. jerk $u_i(t)$) and improve ride quality. Thus, the second objective function is

$$H_2(\mathbf{U}(t)) = \int_{t^0}^{t^f} \left[\sum_{i\in\mathcal{I}} u_i^2(t)\right]^{\frac{1}{2}} dt. \tag{9}$$

Based on the fuel consumption function (6) and comfort function (9), The total cost function of our trajectory model is defined as

$$\min_{u_i} G(\mathbf{U}(t)) = w_1 H_1(\mathbf{U}(t)) + w_2 H_2(\mathbf{U}(t)), \tag{10}$$

where w_1 and w_2 are different weighting coefficients and they reflect the relative importance of the objectives. To obtain a trade-off between the two cost functions $H_1(\mathbf{U}(t))$ and $H_2(\mathbf{U}(t))$, we use a linear weighting multi-objective optimization method to construct a single cost function [42], [43]. According to the different weight values of w_1 and w_2, the fuel economy and comfort of the formation system can be observed, which further helps decision-makers choose different weight coefficients to weigh the two objectives and obtain the corresponding optimal trajectory.

2) Constraints: The constraints of the trajectory planning model include four aspects: Speed constraints; Acceleration constraints; Jerk constraints and Safety constraints. Their mathematical form has the following form:

$$\begin{cases} v^- \leq v_i(t) \leq v^+, i \in \mathcal{I}; & (11) \\ a^- \leq a_i(t) \leq a^+, i \in \mathcal{I}; & (12) \\ u^- \leq u_i(t) \leq u^+, i \in \mathcal{I}; & (13) \\ a_i(t) \leq \min\{a^-, a_j(t)\}, i \in \mathcal{I}, j \in \mathcal{J}. & (14) \end{cases}$$

where v^+, v^-, u^+, u^-, a^+, a^- indicate the upper bounds and lower bounds of the speed, acceleration and jerk. The acceleration $a_j(t)$ is obtained by the OVM model and constraints $a_i(t) \leq \min\{a^-, a_j(t)\}$ shows the smoothly of the handover between the CAVs and HDVs.

Based on the above description, the trajectory planning model for enhancing both fuel economy and ride comfort of the mixed platoon is

$$\begin{aligned} &\min_{u_i}\ G(\mathbf{U}(t)) \\ &\text{s.t.: } (11), (12), (13) \text{ and } (14). \end{aligned} \tag{15}$$

IV. Adaptive control design based on robust tube MPC method

Although optimal trajectories (i.e., ideal acceleration profiles based on fuel consumption and comfort improvements) are generated at the planning layer, real-time operation of CAVs still requires an acceleration adaptation function to cope with unpredictable traffic disturbances during actual operation. For example, the lead CAV of the platoon may be disturbed by uncertain traffic conditions and noise. Therefore, when there is a random disturbance in the system, the controller can effectively adjust the acceleration of CAVs and ensure the safety of formation vehicles.

A. Longitudinal Tracking Dynamics Model for Vehicle Platoon Systems

According to linearization techniques [30] and Zero Order Hold method [44], we transform the time-continuous model into a discrete model by a relatively small time interval τ, e.g., 0.5 seconds. In addition, we adopt a second-order linear model of CAV, which is widely used by [35] and [45]. With the time interval of τ, the discrete dynamic system is described by:

$$\begin{cases} p_i(t+1) = p_i(t) + \tau \cdot v_i(t) + \frac{\tau^2}{2} \cdot a_i(t); \\ v_i(t+1) = v_i(t) + \tau \cdot a_i(t), i \in \mathcal{I}. \end{cases} \tag{16}$$

Assuming that the state and acceleration planned by model (15) are expressed as $\boldsymbol{x}_i^r(t) = [p_i^r(t), v_i^r(t)]^{\mathrm{T}}$ and $a_i^r(t)$, the state deviation of tracking error can be denoted as

$$\begin{cases} e_i^p(t+1) = p_i(t) - p_i^r(t) - r \cdot v_i^r(t) - L; \\ e_i^v(t+1) = v_i(t) - v_i^r(t), i \in \mathcal{I}. \end{cases} \tag{17}$$

where $r \geq \tau$ is a constant representing the reaction time, and $L \geq 0$ is the vehicle length. Then we can deduce

$$\begin{bmatrix} e_i^p(t+1) \\ e_i^v(t+1) \end{bmatrix} = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_i^p(t) \\ e_i^v(t) \end{bmatrix} + \begin{bmatrix} \frac{\tau^2}{2} \\ \tau \end{bmatrix} a_i^r(t) + \begin{bmatrix} -(\frac{\tau^2}{2} + r\tau) \\ -\tau \end{bmatrix} a_i(t). \tag{18}$$

Define the tracking error state variable is $\bar{\boldsymbol{x}}_i(t)$, the dynamic system (18) can be denoted as

$$\boldsymbol{x}_i(t+1) = \boldsymbol{A}\boldsymbol{x}_i(t) + \boldsymbol{B}a_i(t) + \boldsymbol{C}a_i^r(t), \tag{19}$$

where $\boldsymbol{A} = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} -(\frac{\tau^2}{2} + r\tau) \\ -\tau \end{bmatrix}$, $\boldsymbol{C} = \begin{bmatrix} \frac{\tau^2}{2} \\ \tau \end{bmatrix}$. However, deterministic dynamic systems cannot capture the effects of uncertain factors and random disturbances in the actual operating environment. Therefore, a more realistic control model should include uncertain disturbance $\boldsymbol{w}(t)$, adding $\boldsymbol{w}(t)$ to the system (19), we further derive

$$\boldsymbol{x}_i(t+1) = \boldsymbol{A}\boldsymbol{x}_i(t) + \boldsymbol{B}a_i(t) + \boldsymbol{C}a_i^r(t) + \boldsymbol{w}(t), \tag{20}$$

where $\boldsymbol{w}(t)$ is uncertain disturbance during the time step $(t\tau, (t+1)\tau]$ which is defined as the measurement error of the sensor [12]. Assumed that the uncertain disturbances belong to a set $\mathbb{W}$, i.e.,

$$\boldsymbol{w}(k) \in \mathbb{W} = \left\{\mathbb{R}^2 : \|\boldsymbol{w}(k)\|_\infty \leq \Omega\right\}. \tag{21}$$

In order to facilitate the following analysis, we define the nominal tracking error as

$$\bar{\boldsymbol{x}}_i(t+1) = \boldsymbol{A}\bar{\boldsymbol{x}}_i(t) + \boldsymbol{B}\bar{a}_i(t) + \boldsymbol{C}a_i^r(t), \tag{22}$$

and the prediction uncertainty as

$$\tilde{\boldsymbol{x}}_i(t) = \boldsymbol{x}_i(t) - \bar{\boldsymbol{x}}_i(t). \tag{23}$$

In addition, we also need to consider the basic constraints on the state variable $\boldsymbol{x}_i(t)$ and control variable $u_i(t)$ of CAV i. All the constraints are summarized as follows.

$$e^p_{\text{min}} \leq e^p_i(t) \leq e^p_{\text{max}}, \tag{24}$$

among that e^p_{min} and $e^p_{\text{max}}$are the lower and upper bounds of position tracking error for vehicle i.

$$e^v_{\text{min}} \leq e^p_i(t) \leq e^v_{\text{max}}, \tag{25}$$

among that e^v_{min} and $e^v_{\text{mas}}$are the lower and upper bounds of speed tracking error for vehicle i.

$$a_{\text{min}} \leq a_i(t) \leq a_{\text{max}}, \tag{26}$$

among that a_{min} and a_{max} are the lower and upper bounds of acceleration. By the constraints (24) and (25), we can drive the state constraint as follows

$$\boldsymbol{x}^- \leq \boldsymbol{x}_i(t) \leq \boldsymbol{x}^+, \tag{27}$$

where $\boldsymbol{x}^- = [e^p_{\text{min}}, e^v_{\text{min}}]^{\mathrm{T}}$, $\boldsymbol{x}^+ = [e^p_{\text{max}}, e^v_{\text{max}}]^{\mathrm{T}}$. In order to simplify the notation, we define $\mathbb{X}$ and $\mathbb{U}$ as the sets of states and control constraints, respectively, such that $\boldsymbol{x}_i(t) \in \mathbb{X}$ and $a_i(t) \in \mathbb{U}$ holds true for all values of t. Based on Equation (26) and (27), the physical constraints can be integrated as

$$\begin{cases} \mathbb{X} = \{\boldsymbol{x} : \boldsymbol{A}_x\boldsymbol{x} \leq \boldsymbol{b}_x\}; \\ \mathbb{U} = \{\boldsymbol{a} : \boldsymbol{A}_a\boldsymbol{a} \leq \boldsymbol{b}_a\}. \end{cases} \tag{28}$$

where $\boldsymbol{A}_x = \text{col}\{\boldsymbol{I}_{4\times4}, -\boldsymbol{I}_{4\times4}\}$ and $\boldsymbol{b}_x = \text{col}\{\boldsymbol{x}^+, -\boldsymbol{x}^-\}$, respectively. $\boldsymbol{A}_a$ and $\boldsymbol{b}_a$ are defined as $\boldsymbol{A}_a = \text{col}\{1, -1\}$ and $\boldsymbol{b}_a = \text{col}\{a_{\max}, -a_{\min}\}$, respectively.

B. Robust Tube MPC Formulation

In order to resist the influence of uncertain disturbances, we propose a tube-based model predictive control method to adjust the vehicle acceleration obtained by the planning tier. The framework is based on the Tube Model Predictive Control (TMPC) method, widely adopted in the control community due to its ability to handle uncertainties and constraints. The control approach illustrated in Figure (2) comprises two primary elements: feedback control and feedforward control. Through the feedforward control, a sequence of set points, which is called a "tube" and defines the nominal tracking error of the system, is determined. The feedback control, however, dynamically adjusts the actual tracking error, ensuring that it remains within the boundaries set by the tube. This approach allows the system to achieve the desired performance while considering the presence of external disturbances.

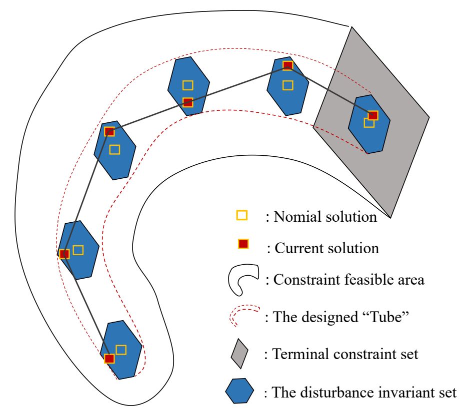


Fig. 2. Illustration of tube MPC method.

For the CAV i, the actual acceleration by

$$a_i^*(t) = \bar{a}_i(t) + \tilde{a}_i(t), \tag{29}$$

at every time step k, $\bar{a}_i(k)$ is determined by the feedforward control and $\tilde{a}_i(k)$ is determined by the feedback control. The feedback control ensures that the actual tracking error remains within the constraints defined by the tube, guaranteeing all constraints' satisfaction.

Specifically, we design a discrete linear quadratic regulator to solve the feedback control. Then, according to constraints (26) and (27), the feasible set is constructed for $\boldsymbol{x}_i(t)$. Using the ϵ-approximation method, the minimum Robust Positive Invariant set (mRPI) is defined and estimated. Finally, the nominal trajectory, or feed-forward control, is determined by computing the optimization problem with a strict feasible set.

1) Feedback Control Law: In order to achieve a closed-loop effect for each controller, the following linear feedback control strategy is used

$$\tilde{a}_i(t) = \boldsymbol{K}\tilde{\boldsymbol{x}}_i(t), i \in \mathcal{I}, \tag{30}$$

where $\boldsymbol{K}$ is gain matrix. The principle of this control strategy is to multiply the error state by the gain matrix $\boldsymbol{K}$ to determine the true feedback control signal of the vehicle i. Furthermore, we employ the concept of rolling optimization to address the model, allowing for the continual updating of the vehicle's state error, $\tilde{\boldsymbol{x}}_i(t)$, with each iteration. For each CAV $i \in \mathcal{I}$ at time t, substituting (30) into (20), we can obtain the following state equation

$$\begin{aligned} \tilde{\boldsymbol{x}}_i(t+1) =& (\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})\tilde{\boldsymbol{x}}_i(t) + \boldsymbol{B}\tilde{a}_i(t) + \boldsymbol{C}a_i^r(t), \\ =& \boldsymbol{A}_K\tilde{\boldsymbol{x}}_i(t) + \boldsymbol{B}\tilde{a}_i(t) + \boldsymbol{C}a_i^r(t) + \boldsymbol{w}(t). \end{aligned} \tag{31}$$

where $\boldsymbol{A}_K = \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K}$. According to the literature [46], we can solve the feedback gain $\boldsymbol{K}$ by a linear quadratic programming model, the model is as follows

$$\min_{\boldsymbol{K}} Y=\sum_{t=0}^{T}\left\{\tilde{\boldsymbol{x}}_i^{\mathrm{T}}(t)\boldsymbol{Q}\tilde{\boldsymbol{x}}_i(t)+\tilde{a}_i^{\mathrm{T}}(t)R\tilde{a}_i(t)\right\}, \tag{32}$$

where $\boldsymbol{Q}=\begin{bmatrix}Q_1 & 0\\ 0 & Q_2\end{bmatrix}$ is a weight matrix that is positive definite and symmetric, and $R>0$.

2) Minimal Robust Positively Invariant Set: The robust positively invariant (RPI) set is defined as a set in which $\boldsymbol{x}_i(t)$ can be bounded by the feedback control in Equation (30). This has been proven in [47].

Definition 1: (RPI set): The set $\mathbb{Z}\subset\mathbb{R}^2$ is a robust positively invariant (RPI) set of the system (31) if $\boldsymbol{A}_K\boldsymbol{x}_i+\boldsymbol{w}\in\mathbb{Z}$ for all $\boldsymbol{x}_i\in\mathbb{Z}$ and all $\boldsymbol{w}\in\mathbb{W}$, i.e., if and only if $A_K\mathbb{Z}\oplus\mathbb{W}\subset\mathbb{Z}$.

As shown in Figure (2), the size of the tube in the blue area is determined by this RPI set. In order to prevent the result from being too conservative, the minimum RPI set can be defined as [47]:

Definition 2: (mRPI set): The minimal Robust Positively Invariant (mRPI) set $\mathbb{F}$ is a subset of the robust positively invariant (RPI) $\mathbb{Z}$ set of (31) that is contained in every closed RPI set of (31).

In general, a specific representation of $\mathbb{F}$ is not available. Based on ϵ-approximation method [48], the equivalently form of mRPI set is $\mathbb{F}=\lim_{s\to\infty}F_s$, where

$$F_s=\bigoplus_{l=0}^{s-1}A_K^l\mathbb{W}, F_0=\{0\}. \tag{33}$$

We obtained the mRPI set $\mathbb{F}$ through the above approximation method, and the feedforward control $\bar{a}_i(t)$ will be designed based on this convex set.

3) Terminal constraints: The feedforward controller is designed to resist the effects of random disturbances by utilizing a convex optimization approach that ensures compliance with the stringent sets derived from tube methods. The purpose of the feedforward control is to find a tube that adheres to all constraints described in Equation (27), where the constraints can be represented by

$$\boldsymbol{x}\in\mathbb{X}, \boldsymbol{a}\in\mathbb{U}. \tag{34}$$

For the nominal tracking error $\bar{\boldsymbol{x}}_i$, it holds under the following constraints

$$\bar{\mathbb{X}}=\mathbb{X}\ominus\mathbb{F}, \bar{\mathbb{U}}=\mathbb{U}\ominus K\mathbb{F}. \tag{35}$$

In order to obtain $\bar{a}_i(t)$, the optimization problem can be formulated as:

$$\begin{aligned}\min_{\bar{a}_i(1),\cdots,\bar{a}_i(N_p)} L=&\sum_{t=1}^{N_p}\left\{\bar{\boldsymbol{x}}_i^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{x}}_i(t)+\bar{a}_i^{\mathrm{T}}(t)V\bar{a}_i(t)\right\}\\ \text{s.t. }&\bar{\boldsymbol{x}}_i(t+1)=\boldsymbol{A}\bar{\boldsymbol{x}}_i(t)+\boldsymbol{B}\bar{a}_i(t)+\boldsymbol{C}a_i^r(t);\\ &\bar{\boldsymbol{x}}_i(t)\in\mathbb{X}\ominus\mathbb{F};\\ &\bar{a}_i(t)\in\mathbb{U}\ominus K\mathbb{F};\\ &\bar{\boldsymbol{x}}_i(N_p)=0;\\ &\bar{a}_i(N_p)=0.\end{aligned} \tag{36}$$

where $t=1,\ldots,N_p$, $\boldsymbol{P}$ and V are weight factors which are symmetric and positive. The construction of the model is based on the prediction principle, in which the choice of prediction time horizon N_p and control time horizon N_c will affect the computational efficiency of the algorithm. On the one hand, it is important to choose a prediction horizon that is large enough to predict control steps for multiple periods accurately. When the prediction period increases, the controller of CAV i can predict longer distances, and its performance improves accordingly. On the other hand, if the prediction time horizon N_p and time horizon N_c are too long, it may increase the calculation time of optimization and the difficulty of realizing control input online. In the experiment section, we will verify the computational performance of the selected prediction and control time domains to ensure the real-time requirements of the formation control system.

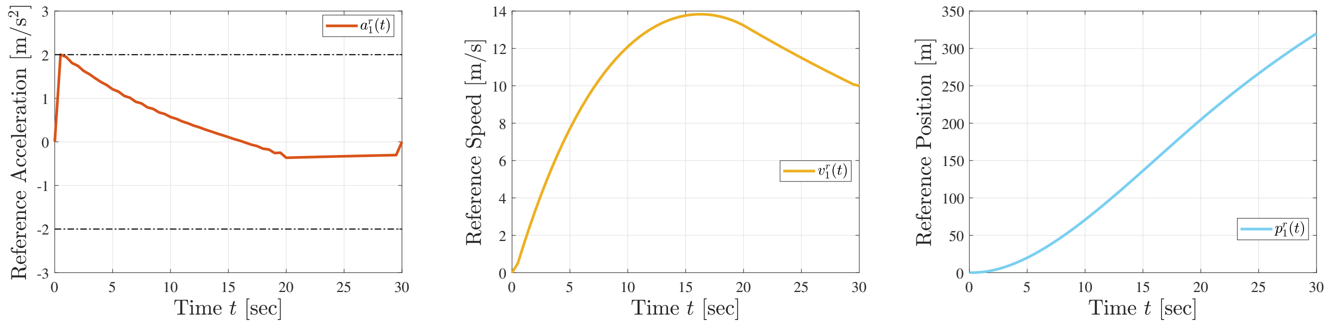
Algorithm 1: The Algorithm Design for Two-tier Optimization

Input: Initial Status; Basic parameters.
Planning tier:
1. Obtain the position p_i^r, speed v_i^r and acceleration trajectory a_i^r of the leading vehicle under the trajectory planning model (15);
2. Define the initial state of every vehicle in the platoon.

Control tier:
1. Obtain the feedback gain $\boldsymbol{K}$ by solving the problem (30);
2. Construct uncertain set $\mathbb{W}$;
3. Obtain the mRPI set $\mathbb{F}$ by ϵ-approximation method and compute the tight constraint sets $\bar{\mathbb{X}}$ and $\bar{\mathbb{U}}$;
4. Real-time control:

while $t\le N_p$ **do**
  Compute the feedforward control $\bar{a}_i(k)$ and obtain the nominal state $\bar{\boldsymbol{x}}_i(t)$ by solving optimization problem (36);
  Compute the feedback control $\tilde{a}_i(t)$;
  Generate the actual control $a_i^*(t)=\bar{a}_i+\tilde{a}_i(t)$;
  Implement the control $a_i^*(1)$ to the system (20);
  Compute the acceleration of HDVs;
  Update $\boldsymbol{x}_i(t+1)$;
  Update $t=t+1$;
end
Output: The real-time control of CAVs and HDVs.

For the following behaviour of HDV, we will solve the OVM model (1) to obtain the acceleration of the vehicle. However, due to the measurement error caused by the sensor in the system, $\Delta p_j(t)$ is also uncertain in the equation (1): $a_j(t)=\eta\left[V_{opt}\left(\Delta p_j(t)\right)-v_j(t)\right], j\in\mathcal{J}$. Therefore assume $\Delta p_j(t)=p_{j-1}(t)-p_j(t)+w_p$, where w_p is uncertain disturbance and $w_p\in[w_{\min},w_{\max}]$. $w_{\min}$ and $w_{\max}$ are the known upper and lower bounds of the disturbance respectively. Based on the worst-case principle [49], [50], we take the lower bound of $\Delta p_j(t)$ as the real value of $\Delta p_j(t)$, i.e., $\Delta p_j(t)=p_{j-1}(t)-p_j(t)-w_{\min}$. This worst-case scenario-based robust control method is also an effective way to resist disturbances and this processing method also makes HDV have a certain degree



(a) Optimal Acceleration (b) Optimal Speed (c) Optimal Position

Fig. 3. The optimal trajectory of the leading CAV at planning tier.

of robustness. Overall, the design of the proposed two-tier optimization algorithm is summarized as **Algorithm 1**.

V. Simulation Results

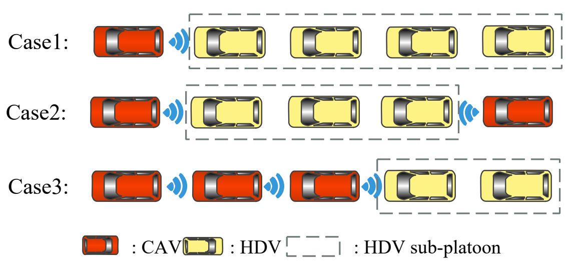


Fig. 4. The platoon configuration in our paper.

In this section, we conduct numerical experiments to verify the performance of the proposed two-tier optimization framework. Firstly, the trajectory of the CAV is obtained based on a bi-objective optimization model, aiming to achieve the best fuel economy and ride comfort at the planning tier. Secondly, we design experiments to validate the effectiveness of the robust tube MPC control algorithm based on the reference trajectory. Third, experimental comparisons are carried out under three different optimization methods. The MATLAB modelling toolbox and YALMIP solve the two-tier optimization model. In addition, Three types of mixed traffic streams are selected for the experiment, as shown in Figure 4. Case 1 and Case 2 represent different positions of CAVs and HDVs in the mixed platoon, while Case 1 and Case 3 represent different penetration rates of CAVs in the platoon. Cases 1, 2, and 3 depict various positions of CAVs and HDVs in the mixed platoon and different penetration rates of CAVs.

A. Experimental Results of CAV (Planning Tier)

The basic parameters set used in the first tier are the following: The minimum and maximum speed $v^- = 0\,\mathrm{m/s}$, $v^+ = 40\,\mathrm{m/s}$; The maximum acceleration and minimum acceleration $a^- = -2\,\mathrm{m/s^2}$, $a^+ = 2\,\mathrm{m/s^2}$; The maximum control and minimum control $u^- = -3\,\mathrm{m/s^3}$, $u^+ = 3\,\mathrm{m/s^3}$; The initial time and terminal time $t^0 = 0\,\mathrm{s}$, $t^f = 30\,\mathrm{s}$.

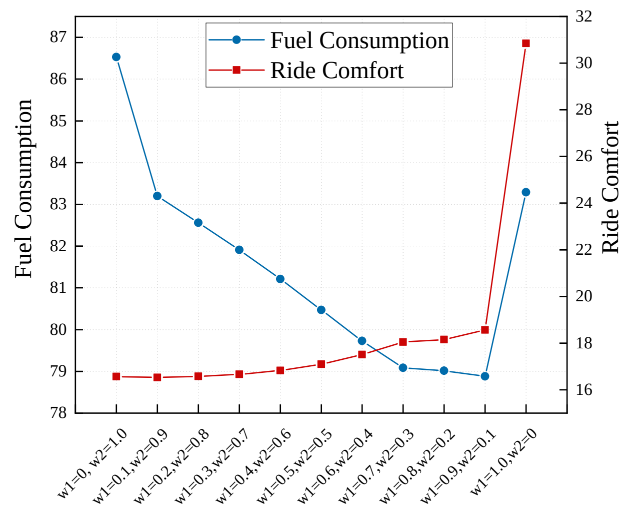
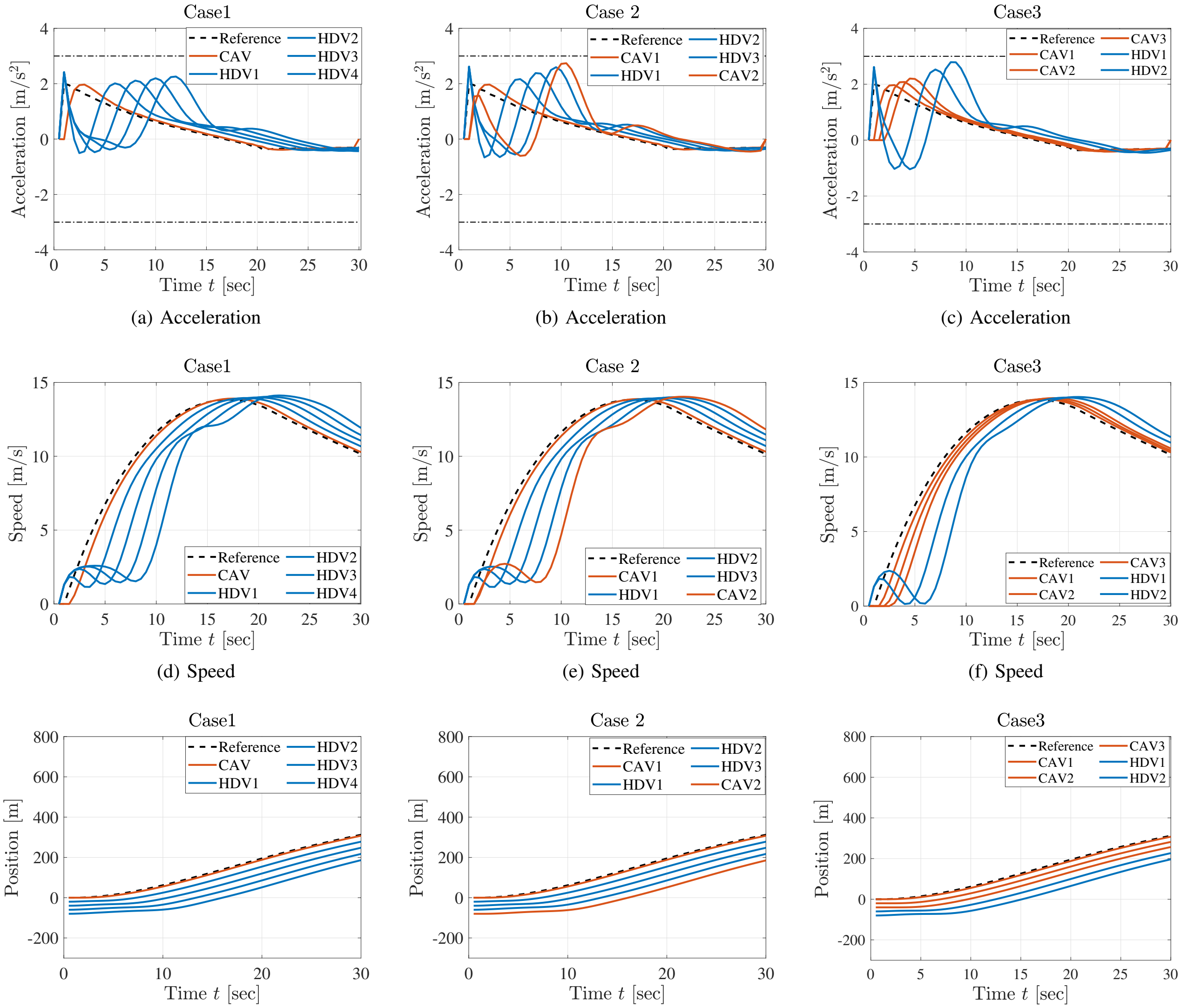


Fig. 5. The objective value under different weights.

TABLE II
Parameters Settings in Control Tier

Parameter	**Value**
τ	$0.5\,\mathrm{s}$
r	$0.5\,\mathrm{s}$
N_p	$2.5\,\mathrm{s}$
N_c	$0.5\,\mathrm{s}$
$a_{\min}$, $a_{\max}$	$-3\,\mathrm{m/s^2}$, $+3\,\mathrm{m/s^2}$
$e^v_{\min}$, $e^v_{\max}$	$-5\,\mathrm{m/s}$, $+5\,\mathrm{m/s}$
$e^p_{\min}$, $e^p_{\max}$	$-2\,\mathrm{m}$, $+2\,\mathrm{m}$
L	$3.5\,\mathrm{m}$
V	1
$\boldsymbol{P}$	$\mathrm{diag}[1, 1]$
$\mathbb{W}_1$	[0.5, 0.5; 0.5, -0.5; -0.5, -0.5; -0.5, 0.5]*0.3
$\mathbb{W}_2$	[1, 1; 1, -1; -1, -1; -1, 1]*0.5

As can be seen from Figure 5, the choice of weighting coefficients w_1 and w_2 represents how decision-makers balance fuel consumption and ride comfort. Therefore, we conducted experiments on different values of w_1 and w_2, where they belong to $[0, 1]$ and $w_1 + w_2 = 1$. Figure 5 illustrates the fuel consumption and ride comfort of the CAV under different objective weightings. From the figure, it can be observed that when we only focus on optimizing the ride comfort of the

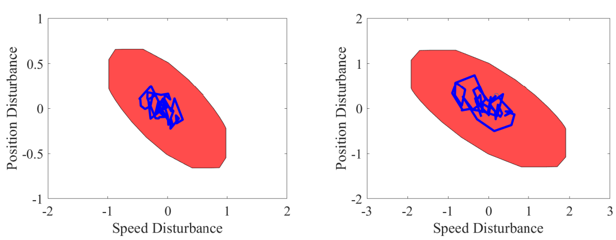


(a) Acceleration (b) Acceleration (c) Acceleration

(d) Speed (e) Speed (f) Speed

(g) Position (h) Position (i) Position

Fig. 6. The results of the mixed platoon under disturbance type I.



(a) Disturbance Type I (b) Disturbance Type II

Fig. 7. The accumulated uncertainty of the platoon system.

vehicle ($w_1 = 0$, $w_2 = 1$), the fuel consumption is maximum, i.e., $86.4973\,\mathrm{mL}$. On the other hand, when only optimizing the fuel consumption objective of the vehicle ($w_1 = 1$, $w_2 = 0$), the ride comfort is poorest, i.e., $30.8429\,\mathrm{m/s^3}$. As can be seen from Figure 5, $w_1 = 0.6, w_2 = 0.4$ is a trade-off point of the system, the fuel economy and comfort objectives are simultaneously optimized, with the fuel consumption objective of $79.7316\,\mathrm{mL}$ and ride comfort of $17.5055\,\mathrm{m/s^3}$. By using the bi-objective optimization model, the fuel consumption of CAV is reduced by 7.85%, and the ride comfort is improved by 5.75%. In addition, the acceleration, speed and position by solving the model (15) are obtained in Figure 3. Figure 3a illustrates the boundary constraints of acceleration with black dashed lines. Based on this, we obtain the optimal acceleration trajectory for improving fuel consumption and ride comfort at the planning tier.

B. Real-Time Robust control of mixed platoon (Control Tier)

To achieve long-term optimization of the mixed vehicle platoon, we apply local adaptation control to the real-time trajectory of the vehicles to cope with the stochastic disturbances existing in the traffic environment. At the control tier, we consider three specific mixed vehicle platoon systems that consist of CAVs and HDVs in Figure 4, and the reference trajectory (indexed from 0) of the CAV is obtained at the planning tier (Figure 3). For the control layer, the initial speed

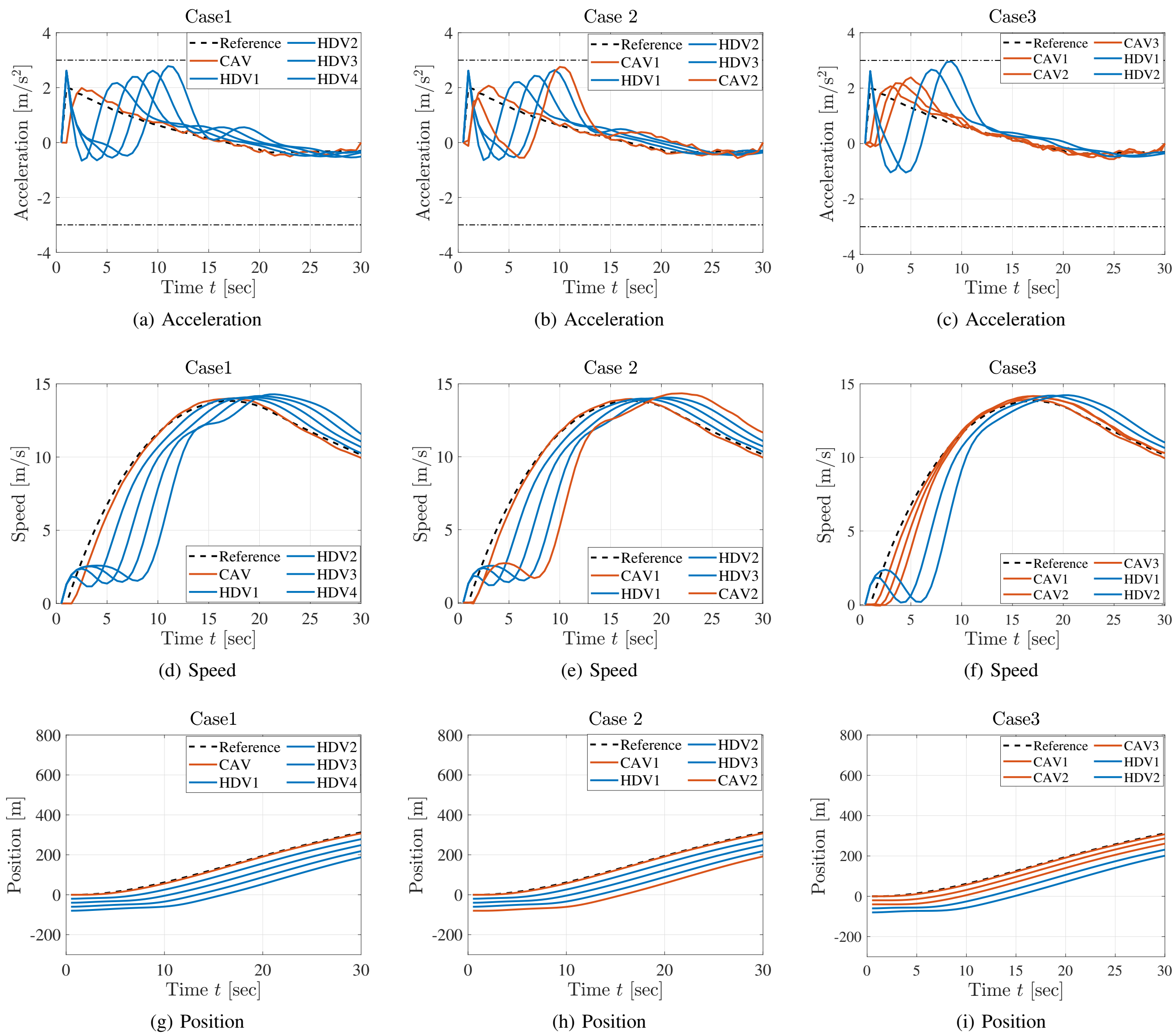


Fig. 8. The results of the mixed platoon under disturbance type II.

TABLE III
THE OBJECTIVES UNDER THREE CASES

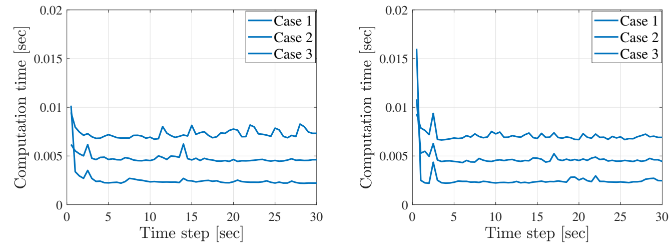
	Disturbance Type I		Disturbance Type II	
	Obj1 [mL]	Obj2 $[\mathrm{m/s^3}]$	Obj1 [mL]	Obj2 $[\mathrm{m/s^3}]$
Case 1	479.2138	139.0117	479.5717	143.2312
Case 2	475.5198	137.2966	474.9894	**141.6586**
Case 3	**470.2836**	**130.2966**	**474.6346**	148.7766

of all vehicles is $0\,\mathrm{m/s}$. The initial longitudinal positions are generated in the range [-200,0] m, and the distance between vehicles is at least $20\,\mathrm{m}$. For the HDVs, the value of w_p belongs to $[-0.1, 0.1]$m. Other parameters are set in Table II.

To demonstrate the advantages of the proposed robust tube-based model predictive control method, we conduct simulation experiments under different types of random disturbances. Specifically, we chose two convex sets of different sizes as shown in subfigures (7a) and (7b), and simulated two types of external disturbances under each convex set. Figures 6 and 8 depict the trajectories of mixed vehicle platoons under two types of disturbances. Specifically, Figure 6 presents the acceleration, velocity, and position of three types of mixed platoons under small-scale random disturbances using the robust tube control method. To facilitate visualization, the acceleration trajectories of all platoons of vehicles are shown. In subfigures (6a)-(6c), the black dotted line represents the reference acceleration obtained from the planning tier, while the red curve denotes the real-time control acceleration of CAV. It is evident that the CAV promptly responds to uncertain disturbances in the system under the robust tube control approach, and its acceleration is strictly constrained within the designated range [-3 $\mathrm{m/s^2}$, 3 $\mathrm{m/s^2}$]. Moreover, as shown in Figures 6d-6i, each HDV can track the velocity of its preceding vehicle. Despite the presence of external disturbances, all vehicles can form a stable platoon without collisions. Similarly, Figure 8 displays the trajectory of the mixed platoon subject to large-scale random perturbations. The mixed vehicle platoon in all three cases exhibits robustness to uncertain disturbances, limiting acceleration strictly within the specified range and maintaining a stable formation. Additionally, Table III provides an overview of the fuel economy and ride comfort objectives for the mixed vehicle platoon under two types of disturbances. The results reveal that different types of vehicle platoons have different objectives, and the mixed vehicle platoon system in Case 3 achieves the lowest fuel consumption and the highest comfort level. Experimental results can help decision-makers choose appropriate platoon configurations in different environments.

C. Computation performance of ***Algorithm 1***

Furthermore, we evaluate the computational performance of **Algorithm 1** under two perturbation types. In Figure 12, we show the execution time of three platoon cases under two disturbance types. It can be seen from the figure that when $N_p = 2.5\,s$, the execution time under **Algorithm 1** does not exceed the control horizon $N_c = 0.5\,s$. For instance, the average computation times of the three formation systems under large-scale random disturbances are 2.5 ms, 4.7 ms, and 7.29 ms, which are much smaller than 0.5 s. Therefore, the proposed two-tier optimization algorithm can control vehicle operation in real-time.

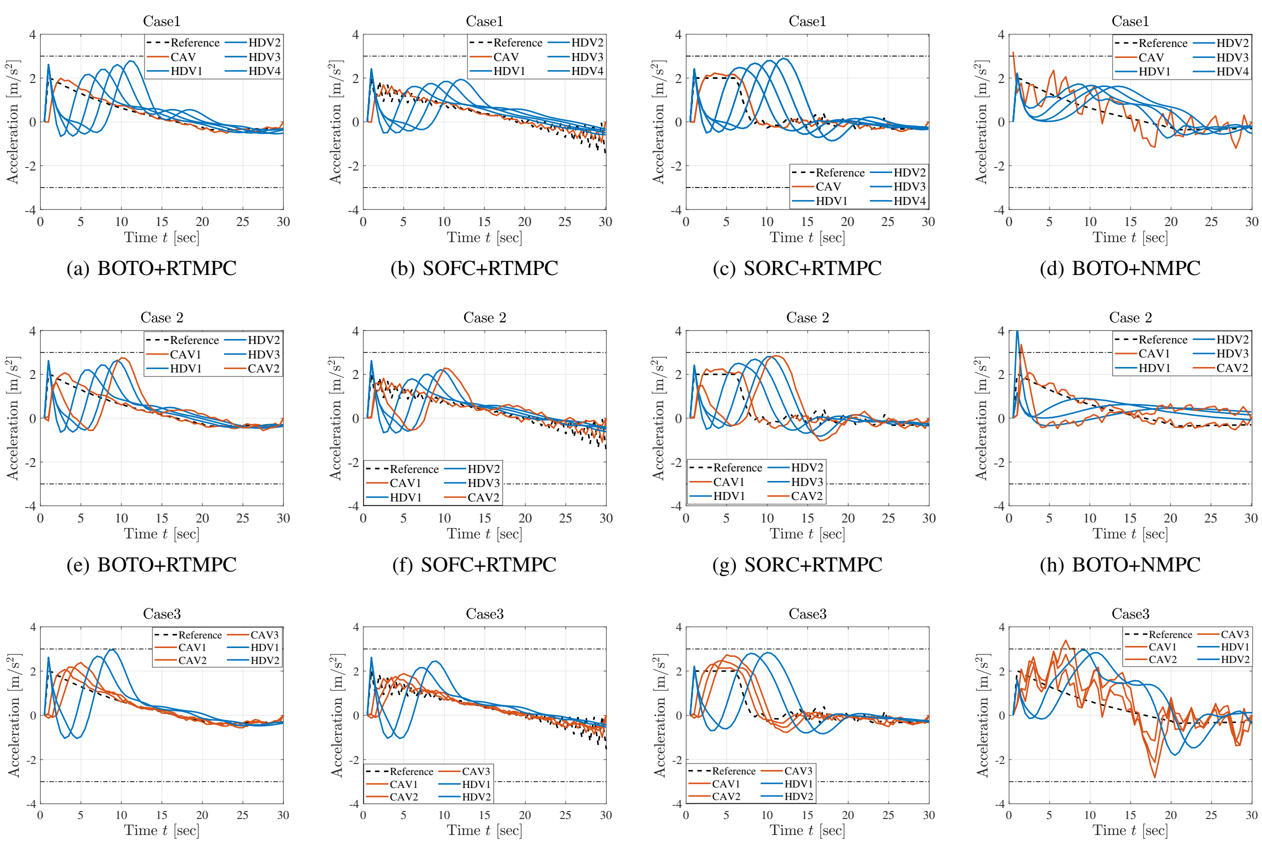


(a) Execution time under I (b) Execution time under II

Fig. 12. Execution time of three cases under two disturbance types.

D. Comparison Experiment

To assess the efficacy of the proposed dual-tier optimization approach and enhance the trustworthiness of simulation outcomes, we undertake a comparative analysis involving three baseline strategies. Firstly, to capture the fuel efficiency and ride comfort aspects of the proposed method, we compare it against two other single-objective methods. Secondly, in order to gauge the disturbance resilience of the RTMPC controller, we compare it to the nominal MPC (NMPC) under the assumption of known optimal vehicle trajectory. Additionally, we conduct experiments across three distinct scenarios (CASEs) to investigate the influence of CAV penetration on formation effects. The three benchmark strategies are described as follows.

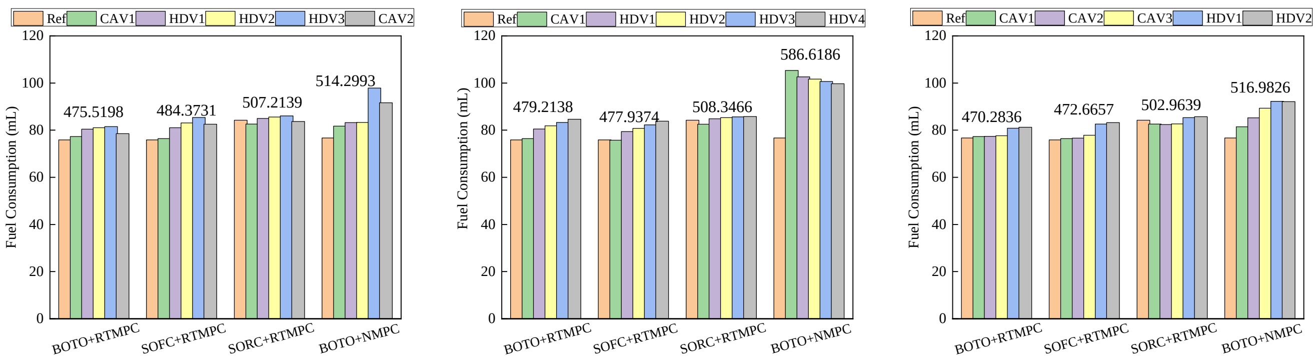


(a) BOTO+RTMPC (b) SOFC+RTMPC (c) SORC+RTMPC (d) BOTO+NMPC

(e) BOTO+RTMPC (f) SOFC+RTMPC (g) SORC+RTMPC (h) BOTO+NMPC

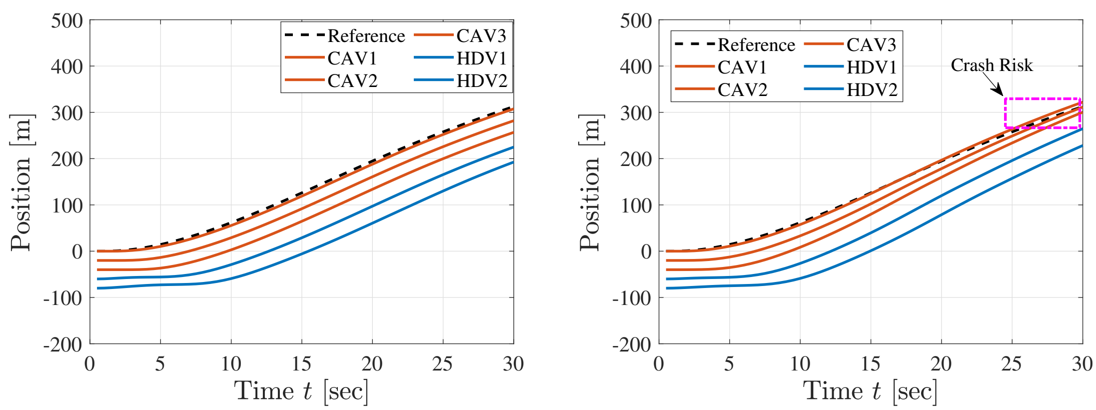
(i) BOTO+RTMPC (j) SOFC+RTMPC (k) SORC+RTMPC (l) BOTO+NMPC

Fig. 9. The comparison of ride comfort under different methods (the smoother the better).



(a) Case1 (b) Case2 (c) Case3

Fig. 10. The comparison of fuel economy under different methods (the smaller the better).



(a) The position of vehicles under BOTO+RTMPC (b) The position of vehicles under BOTO+NMPC

Fig. 11. The comparison of platoon safety under different methods.

- **BOTO+RTMPC**: The proposed two-tier optimization strategy presents a bi-objective trajectory optimization model at the planning tier to improve the fuel economy and ride comfort of the mixed vehicle platoon and designs a robust tube MPC controller at the control tier to optimize the vehicle acceleration in real-time to resist uncertain disturbances.
- **SOFC+RTMPC**: At the planning tier, we obtain the reference trajectory by optimizing the single-objective fuel consumption model and then design a robust tube controller in the control layer to optimize the vehicle acceleration in real time to resist uncertain disturbances.
- **SORC+RTMPC**: At the planning tier, we solve the single-objective ride comfort model to obtain the reference trajectory and then design a robust tube controller in the control layer to optimize the vehicle acceleration in real time to resist uncertain disturbances.
- **EDO+NMPC**: At the planning tier, the reference trajectory of the vehicle is obtained based on the eco-driving model, and the nominal MPC is used to control the CAVs to adjust the speed in real time. There is no ability to cope with uncertainty in this strategy.

Subsequently, we compare the four methodologies across three key facets: ride comfort, fuel efficiency, and formation safety (pertaining to disturbance resilience) in the presence of minor disturbances. As described in [2], frequent acceleration/deceleration can lead to discomfort for passengers and drivers alike. Hence, we visually present the accelerations associated with the four approaches in three cases, as depicted in Figure 9, to facilitate an experiential assessment of the system's comfort within the formation and the fuel consumption of the vehicle platoon under different methods can be shown in Figure 10. As illustrated in Figures 9 and 10, the proposed BOTO+RTMPC method exhibits superior performance in terms of both vehicle economy and comfort when compared to the two single-objective optimization methods, namely SOFC+RTMPC (solely focused on fuel consumption) and SOCR+RTMPC (solely focused on comfort). Notably, the BOTO+RTMPC method strikes a commendable balance between these performance aspects, reducing fuel consumption for the entire formation system while maintaining a relatively comfortable driving environment. Notably, the BOTO+RTMPC method strikes a commendable balance between these performance aspects, reducing fuel consumption for the entire formation system while maintaining a relatively comfortable driving environment. Specifically, from subfigures (9f) and (10b), when compared to the SOFC+RTMPC method (solely focused on fuel consumption), the proposed method substantially enhances the overall vehicle comfort experience, even though there is a slight increase in fuel consumption. Moreover, in comparison to the SORC+RTMPC method, the proposed approach achieves a remarkable 5.73% reduction in fuel consumption while simultaneously ensuring ride comfort, as demonstrated in Case 2. Furthermore, we examined the impact of varying CAV penetration rates on the performance of the formation vehicles. As can be observed from Figure 12, when the penetration of CAVs is higher (case 3), the fuel consumption of the whole formation system is smaller. Overall, the experimental results show that the proposed BOTO+RTMPC method ensures formation comfort while simultaneously reducing the overall system fuel consumption.

To assess the robustness of the proposed controller, we conduct a comparison between the RTMPC controller and NMPC under the same optimal trajectory. As depicted in Figure 11, we show the position of the vehicles in the platoon under BOTO+RTMPC and BOTO+NMPC methods in the presence of minor disturbances. With the BOTO+RTMPC method, all vehicles successfully adhere to the safety constraints, ensuring a safe driving environment. Conversely, with the BOTO+NMPC method, which lacks robust disturbance mitigation capabilities, the inter-vehicle distance is getting smaller and smaller, thereby posing a risk of collisions during driving. Overall, our two-tier optimization model achieves optimal trajectory planning by considering fuel consumption and ride comfort at the planning tier. Additionally, we incorporate robust adaptive control at the control tier to address uncertain disturbances effectively. Our experimental results demonstrate that the proposed two-tier framework can effectively manage uncertain disturbances while maintaining the fuel economy and ride comfort of the mixed vehicle platoon. However, it's important to acknowledge potential limitations related to the robustness of our approach. Factors such as communication interruption and hardware failures may impact the performance of our system in real-world scenarios, such as Network congestion and hardware malfunction. Despite these challenges, we remain committed to refining our approach and addressing these limitations to enhance the reliability and robustness of our system in practical applications.

VI. Conclusion and Future work

A multi-objective control framework for a mixed-vehicle platooning system was proposed in this paper. The main objectives of the framework were to achieve minimum fuel consumption, maximum passenger comfort, and robustness of the platoon. At the offline planning tier, the optimal trajectory was generated for the platoon of hybrid vehicles based on their future routes and driving characteristics, which is aimed at optimizing both fuel economy and passenger comfort. Moreover, we adopted OVM to describe and predict the trajectory of HDVs in the platooning system. In the real-time control tier, a robust tube-based controller based on distributed MPC was designed to address the uncertainties and interferences in the platoon system. The controller can dynamically adjust the acceleration of each vehicle to improve the robustness and maintain the stability of the platoon. Finally, extensive simulation experiments were carried out to illustrate the efficacy of the proposed two-tier optimization framework in reducing fuel consumption, improving passenger comfort, and ensuring platoon safety. In the future, we will explore several research directions, including 1) verifying the performance of the proposed mixed vehicle platoon system through field testing; 2) investigating the communication security issues of the system, and 3) developing prediction models for HDVs further to improve the control performance of the platoon system.

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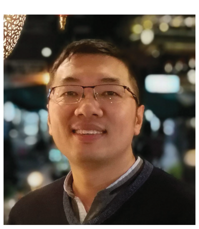

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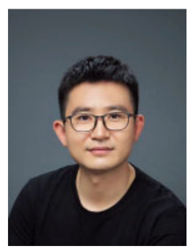

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